Asymptotic Shape of Quantum Semigroup for Quasi-Compact Markov Superforest

An Exploration of Biology in Complexity

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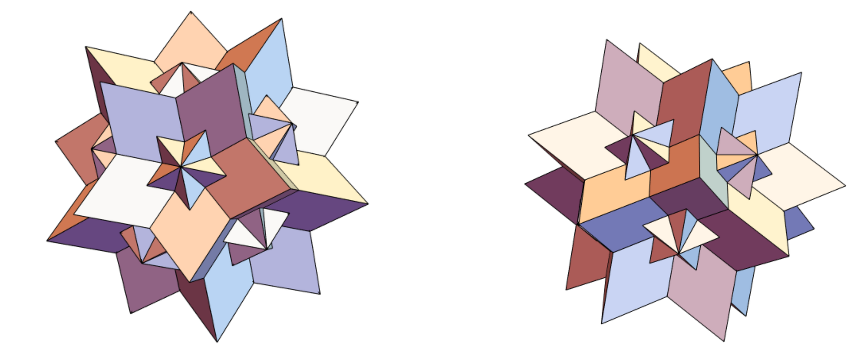

Abstract

Genus $g\gg 0$, supersymmetric process $[0,\infty)\times\mathbb{R}^{|\mathcal{A}^{\otimes n}|}$ of L^p-compact, separable Hilbert, locally finite Markov X superforest adapts quantum $\mathrm{U}(|\mathcal{A}^{\otimes n}|)$ semigroup, conditional-, uniform-partition metric. All $2^{2^{\aleph_0}}$ measurable full group $\mathrm{Aut}(\mathcal{T})$ of modular subgroups grows ergodic-asymptotic shape of Bernard et al. (2019) constant complexity $\mathcal{O}(1)$.

Markov superforest quantum quasi-compactness

Adhering L^p compact $(\Omega,\mathcal{F},\mathbf{P})$ form $\Phi_{\mathcal{X}}$ Markov graph, $\mathcal{X}$-acting process $(X_t)_{t\geqslant 0}:=[0,\infty)\times\mathbb{R}^{|\mathcal{A}^{\otimes n}|}$, the main result is $|\mathcal{A}^{\otimes n}|^{-1}\,\|\mathbf{T}-\mathbf{T}^*\|_{\mathcal{L}(L^p(\gamma))}\leqslant\varepsilon$, uniform weakly (ε), affine bilinear mapping $\gamma\colon\mathcal{P}\longrightarrow\Phi$, $\Phi\colon\mathbb{R}^{|\mathcal{A}^{\otimes n}|}\longrightarrow\mathbb{R}_+$, convergence, for all (superforest) X subforest $\mathfrak{X}\subseteq X$, by

$$\left.\begin{aligned}
\Phi_X &= \sum_x \mathbf{P}_x\, tr(x^\dagger x\ \mathbf{\Gamma}_x)\\
&\quad\text{resp.}\ \int_{\overline{\mathbb{R}}^{|\mathcal{A}^{\otimes n}|}} \det\Big(\sum_x x^\dagger x\ \exp\big(-iH_{\mathbf{\Gamma}}(\theta_x)\big)\Big)\,\mathbf{P}_t\big(\vec{\theta_x}\in d\vec{\theta_x}\big)\\
\mathbf{T}_X &= \sum_x \mathbf{P}_x\, x^\dagger x\\
&\quad\text{resp.}\ \sum_x x^\dagger x\ \exp\big(-\delta(x)\big)\,\mathbf{P}_t\big(\vec{\theta_x}\in d\vec{\theta_x}\big)\\
&\overline{\mathbb{R}}=\mathbb{R}\cup\{-\infty,+\infty\};\ \ x\in\big(\mathcal{A}^{\otimes n}\cup\mathbf{I}_{|\mathcal{A}|^n}\big);\ \ x^\dagger=\overline{x^{\mathrm{T}}};\ \ m,n,p\in\mathbb{N}\\
&x_\xi=x_{\xi_0}\cdots x_{\xi_{m-1}}\equiv x_{\xi_0}\otimes\cdots\otimes x_{\xi_{m-1}};\ \ \mathbf{P}_x=\mathbf{P}_X(x)=\mathbf{P}(X=x)\big|_{x\in\mathcal{X}}
\end{aligned}\right\}\tag{1}$$

on all $2^{2^{\aleph_0}}$ manifold measurable real subforest $(\mathfrak{X}_{\mathbf{T},\mathbf{T}^*},\varepsilon)$, stopping time t, semigroup $(\mathbf{P}_t)_{t\geqslant 0}$, bipartite conditional states $\{\mathbf{\Gamma}_{Q|X=x}\}_{x\in\mathcal{X}}$ operator

$$\left.\begin{aligned}
\mathbf{\Gamma}_{XQ} &= \sum_x \mathbf{P}_x\, x^\dagger x\otimes\mathbf{\Gamma}_{Q|X=x}\\
&\quad\text{resp.}\ \sum_x x^\dagger x\ \exp\big(-iH_{\mathbf{\Gamma}_{Q|X=x}}(\theta_x)\big)\,\mathbf{P}_t\big(\vec{\theta_x}\in d\vec{\theta_x}\big)
\end{aligned}\right\}\tag{2}$$

where Dirac-delta $\delta(x)$ gets the Lagrangian guage point, zero Hamiltonian, in the quantum gravity; for semi-definite H set $\mathcal{P}$ of density (normalized), subnormalized states:

$$\{\mathbf{T}\colon tr(\mathbf{T})=1\}\subseteq\mathcal{P},\quad\text{resp.}\quad\{\mathbf{T}\colon tr(\mathbf{T})\leqslant 1\}\subseteq\mathcal{P}\tag{3}$$

i.e. the classical which is the diagonal $\mathbf{T}$ of i.i.d. eigenfunctions $\mathbf{P}_x$, resp. $\mathbf{P}_t$, on ν-partite Hilbert space $\mathcal{X}_{X_0\cdots X_{\nu-1}}=\mathcal{X}_0\oplus\cdots\oplus\mathcal{X}_{\nu-1}$ supersymmetric basis $\mathcal{A}^{\otimes n}$, real Lie unitary group $\mathrm{U}(|\mathcal{A}^{\otimes n}|)$ uniquely affine functional:

$$\mathbf{\Gamma}\longrightarrow\Phi_{\mathbf{\Gamma}}(F)\mid\mathbf{\Gamma}_{X_\xi}=\mathrm{tr}_{X_\xi}\big(\mathbf{\Gamma}_{X_0\cdots X_{\nu-1}}\big),\ \Phi_{\mathbf{\Gamma}_x}(F)\neq\Phi_{\mathbf{\Gamma}_y}(F)\big|_{x\neq y},\ F\in\mathcal{F}\tag{4}$$

for all superforest domain $\mathcal{X}$ reduced states $\mathbf{\Gamma}_{X_0},\ldots,\mathbf{\Gamma}_{X_{\nu-1}}$, unique mixed state $\mathbf{\Gamma}_{X_0\cdots X_{\nu-1}}\in\mathcal{P}_{X_0\cdots X_{\nu-1}}$, mixing smallest field $\{X^{-1}(B)\colon B\in\mathbb{B}(\mathbb{R}^{|\mathcal{A}^{\otimes n}|})\}$ with $\mathbb{R}^{|\mathcal{A}^{\otimes n}|}$ open sets (intervals) generated Borel sigma field $\mathbb{B}$ such that events $\mathbb{X}\ni x\in\mathcal{X}$ measurable subsets $\mathbb{X}\subset\mathcal{X}$ form sigma field $\mathcal{F}(\mathcal{X})$ of measurable space $(\mathcal{X},\mathcal{F})$ for all outcome.

Markov superforest quantum semigroup existence

Lemma 1.1 (limit semigroup). *Density-, eigen-distance* $\|\mathbf{T}-\mathbf{T}^*\|_{\mathcal{L}(L^p(\gamma))}$, $|\lambda-\lambda^*|$, *i.i.d.-increment* $W(t_1)-W(0),\ldots,W(t_k)-W(t_{k-1})|_{k\in\mathbb{N}}$ *subforest,* $\Phi_{\mathcal{X}}$ *sample functions* $t\longmapsto W(t,\cdot)$ *semigroup* $(\mathbf{P}_t)_{t\geqslant 0}$ *exists in uniform limit.*

Proof. For $|\mathcal{A}^{\otimes n}|$-block ($\mathbb{S}^{|\mathcal{A}^{\otimes n}|-1}$-ball) time $[0,1]$ intervals $f_{\eta\geqslant 0}\geqslant\xi\in\mathbb{N}$, state space $\mathcal{X}$ point $(0,\ldots,0)$ starting $|\mathcal{A}^{\otimes n}|$-tuple space $\mathcal{C}[0,1]$ of continuous functions for all 0-starting $|\mathcal{A}^{\otimes n}|$ paths planar random walk ensemble, take $t\in\mathbb{R}_{\geqslant 0}$, $\frac{\eta}{2}\in\mathbb{N}_0$, equivalence-classes glueable interpolation by

$$\mathfrak{X}_\eta=\left\{\frac{t}{(\frac{\eta}{2})!\,2^{(\eta/2)}}\colon 0\leqslant t\leqslant f_\eta=\left(\frac{\eta}{2}\right)!\,2^{(\eta/2)}\right\}\ \Big|\ \mathfrak{X}=\bigcup_{\eta=0}^{\infty}\mathfrak{X}_\eta\tag{5}$$

where $W_\xi(0)=0$, $W_\xi(1)=Z_1^{(\xi)}$; $\xi=1,\ldots,f_\eta$; $W(t)=(W_1(t),\ldots,W_\eta(t))^T$; $Z_t=(Z_t^{(1)},\ldots,Z_t^{(\eta)})^T$; $Z_{t_\eta^{k_\eta}}\cong\mathcal{N}(0,1)$; $t_\eta^{k_\eta}\in\mathcal{T}_\eta\backslash\mathcal{T}_{\eta-1}$; $k_\eta\in(1,\ldots,2^\eta)$, for

$$W(t_\eta^{k_\eta})=\frac{1}{\eta!}\left(W\Big(t_{\eta-1}^{k_{\eta-1}}+\frac{1}{f_{\eta-1}}\Big)+W\Big(t_{\eta-1}^{k_{\eta-1}}\Big)\right)k_\eta+\frac{1}{f_\eta}Z_{t_\eta^{k_\eta}}.\tag{6}$$

Clearly, of first summand exponential, $W\colon[0,1]\longrightarrow\mathbb{R}^{|\mathcal{A}^{\otimes n}|}$ is i.i.d.-increment affine random process in *uniform convergence threshold*. Then set $\mathbf{P}_t f|_{t\geqslant 0}$; $f\colon\mathbb{R}^{|\mathcal{A}^{\otimes n}|}\longrightarrow\mathbb{R}_+/(\mathcal{A}^{\otimes n}\cup\{\mathbf{I}_{|\mathcal{A}^{\otimes n}|}\})$; $\mathbf{P}_0 f=f$; where in $\mathbf{P}_t^{\otimes n}$ convergence $\mathbf{P}_t$:

$$\lim_{n\uparrow\infty}|\mathcal{A}^{\otimes n}|^{-1}\,\|\mathbf{P}_t^{\otimes n}(*)-\mathbf{P}_t(*)\|_{\mathcal{L}(L^p(\gamma))}\longrightarrow 0,\quad\forall\,p\in[1,\infty].\tag{7}$$

That is, the claim [1]: Φ Feynman path amplitudes $(\mathbf{P}_t)_{t\geqslant 0}$ as locally compact genus $g>0$ single pole removable singularity meromorphic extension family for p-ary rings (p-adic integers p-series field $\mathbb{F}_p$) formal Laurent enumerative $(X_t)_{t\geqslant 0}$ Hadamard completion in entire $\rho(z)=e^{P_n(z)}=\int_\Omega c(zx)\,\mu(dx)$, finite state space $V\cong(0,1,\ldots,n-1)$ irreducible (periodic transition) Markov chain $(V,\mathbf{P})$ subforest isometry braid group action constellation: Fig. 1 and 2. □

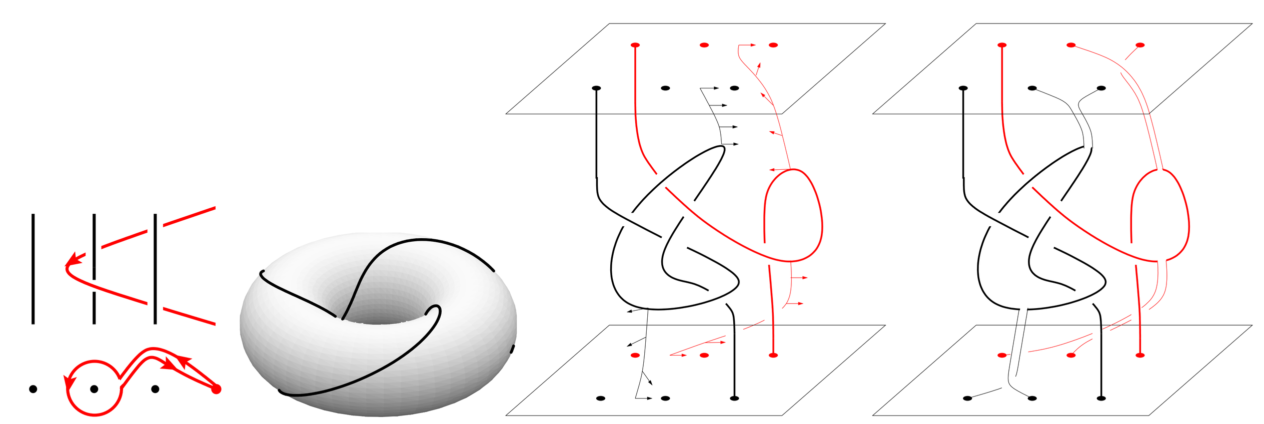

Fig. 1: Subforest isometry semigroup $\mathbb{S}^1\longmapsto\mathbb{R}^3$ braids, string-links, winding compositions.

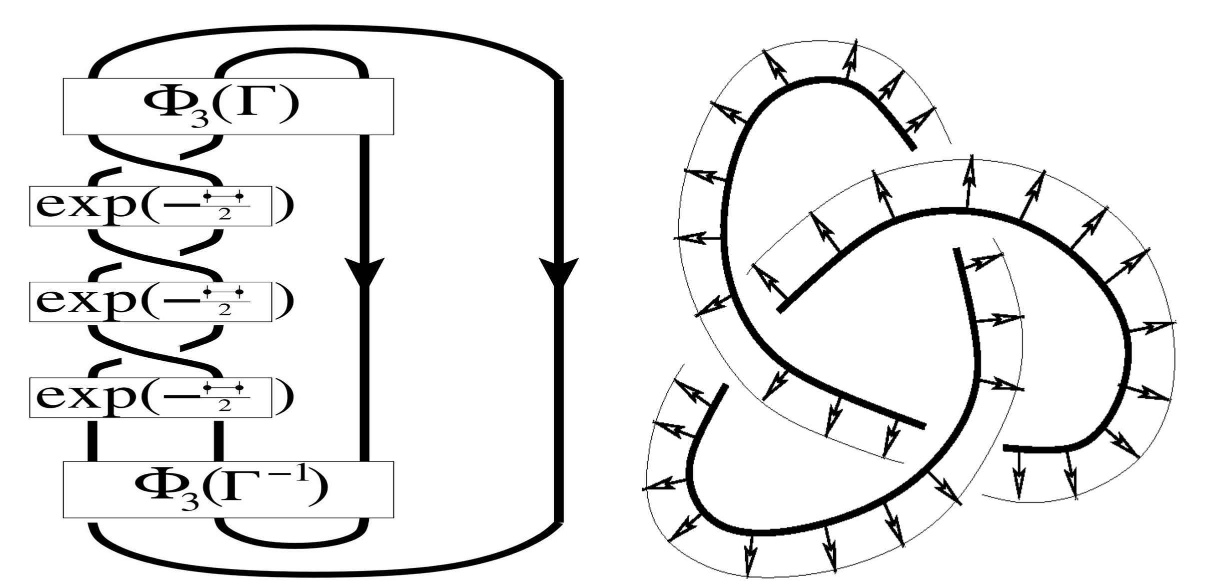

Fig. 2: Left: Subforest knot partition. Right: Dual framed-knot spinor for cohomologous space $\{\mathcal{H}^1(X;\mathbb{Z}_2)\}$ of holomorphic (analytic) in zdz polynomial $p(z)$ quadratic form.

Remark 1.1. In Alice, Bob, and listener Eva known $\{\mathbf{P}_x\}_{x\in\mathcal{X}}$ constellation: $\frac{1}{2}\sum_{x\in\mathcal{X}}|\mathbf{P}_x-\mathbf{P}_x^*|\leqslant\varepsilon$ implies uncorrelated $\mathbf{P}[x\neq x']\leqslant\varepsilon$ for uniform $\{\mathbf{P}_{x'}^*\}$.

Results

Table 1: Illustrated Φ computation for $\mathbf{P}_0=\cdots=\mathbf{P}_n=(n+1)^{-1}$.

$\widetilde{\mathbf{T}}_\xi$	$=\lvert x_{\xi_0}\otimes\cdots\otimes x_{\xi_{m-1}}\rangle\langle x_{\xi_0}\otimes\cdots\otimes x_{\xi_{m-1}}\rvert=(\delta_{k\ell})\rvert_{k,\ell=0}^{n-1}$; $\delta_{k\ell}=\begin{cases}1 & \text{if } \xi=k=\ell\\ 0 & \text{otherwise}\end{cases}$ i.e. the projection operator into eigenvalue $\lambda_{\xi_0\cdots\xi_{m-1}}$ eigenspace.
$\widetilde{\mathbf{T}}_n$	$=n^{-1}\,\mathbf{I}_n=n^{-1}\sum_{\xi=0}^{n-1}\widetilde{\mathbf{T}}_\xi$ i.e. the mixed state.
$\mathbf{T}$	$=n^{-1}\,\mathbf{I}_n=\sum_{\xi=0}^{n}\mathbf{P}_\xi\widetilde{\mathbf{T}}_\xi$; $\quad\mathbf{T}^{-1/2}=\sqrt{n}\,\mathbf{I}_n$
$\mathbf{\Gamma}_\xi$	$=n(n+1)^{-1}\widetilde{\mathbf{T}}_\xi=\mathbf{T}^{-1/2}\,\mathbf{T}_\xi\,\mathbf{T}^{-1/2}$; $\ \xi=0,\ldots,n-1$; i.e. reduced states.
$\mathbf{\Gamma}_n$	$=(n+1)^{-1}\mathbf{I}_n=n^{-1}n(n+1)^{-1}\mathbf{I}_n=n^{-1}\sum_{\xi=0}^{n-1}\mathbf{\Gamma}_\xi$; $\quad\mathbf{I}_n=\sum_{\xi=0}^{n}\mathbf{\Gamma}_\xi$
$\Phi_{\mathbf{\Gamma}}$	$=(n+1)^{-2}(n^2+1)\frac{1}{n}\sum_{\xi,\eta=0}^{n-1}\mathbb{1}_{\xi\xi}\big((\delta_{\xi\eta})\big)=\sum_{\xi=0}^{n}\mathbf{P}_\xi\,\mathrm{tr}(\widetilde{\mathbf{T}}_\xi\mathbf{\Gamma}_\xi)=\mathbb{E}^{gen}[\mathbf{\Gamma}]$

Note: $|\mathcal{A}|^m+1$ quantum states of $|\mathcal{A}|^m+1$ mixture $\Phi_{\mathcal{X}}$ including mixed state.

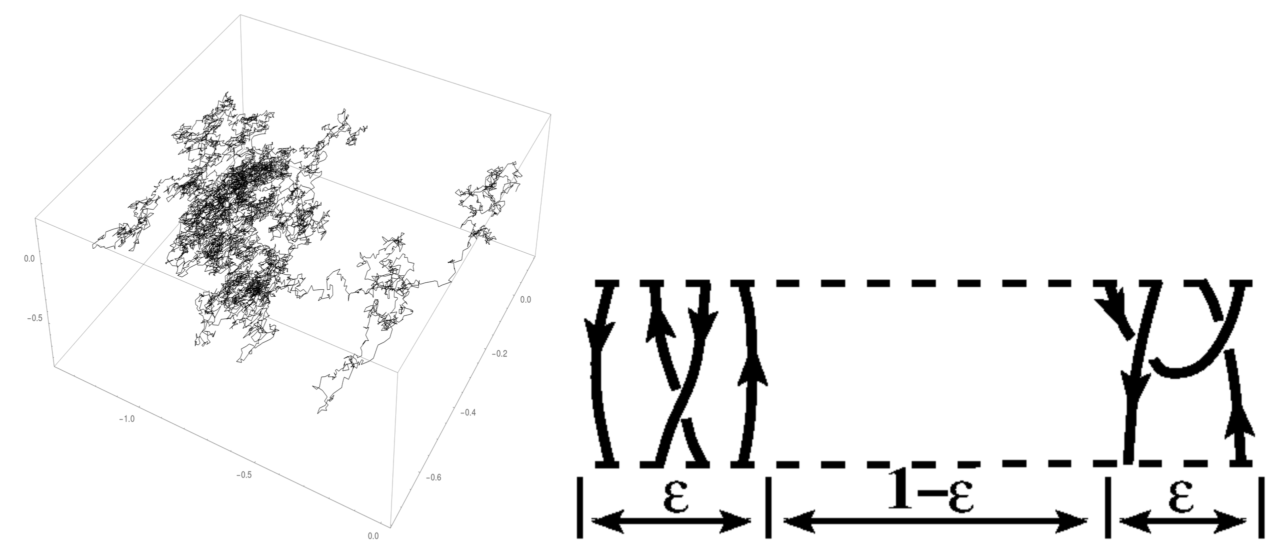

Fig. 3: Left: $\mathcal{C}[0,1]$ of i.i.d.-increment, $(0,0,0)$-starting 3-block ($\mathbb{S}^2$ ball) Markov subforest. Right: ε-parameterized $\mathbb{S}^1\longmapsto\mathbb{R}^3$ in $T_1\otimes_\varepsilon T_2$ inter-tangle isometry ε-rescaling $[z,t]\longmapsto[\varepsilon z,t]$.

Theorem 1.1 (classical bipartite metrizing). *Let compact separable Hilbert* $L^p(\gamma)$ *stopping time* t *subforest* $(X_t)_{t\geqslant 0}$, *unique* $\mathbf{P}_X(x)|_{x\in\mathcal{X}}$ *mixed state,* $|\xi|=n+1$ *mixture, joint-uniform, -conditional density* $\mathbf{T}_{KG}^*$, $\mathbf{T}_{KG}$, *bipartite space* $\mathcal{X}_K\oplus\mathcal{X}_G$, *family* G *of* $|\mathcal{A}|$*-universal hash functions* $\kappa\in K=G(X)$ *have minimum (ergodic, non-ergodic) entropy* $h_{\min}$, $\forall\,g\in G$, *by*

$$g\colon\mathcal{A}^{\otimes m}\cong\mathcal{X}\longrightarrow\mathcal{A}^{\otimes k}\ \mid\ \mathcal{A}^{\otimes\zeta}\cong G\tag{8}$$

then for all $h_{\min}\geqslant h_+$ *low enough, strong protocol of indistinguishable* κ,

$$\frac{1}{|\mathcal{A}|}\lim_{n_t\uparrow\infty}\frac{1}{n_t}\big(\|\mathbf{T}_{KG}-\mathbf{T}_{KG}^*\|_{\mathcal{L}(L^p(\gamma))}\big)^{\otimes n_t}\ \leqslant\ \underset{(0,1]}{\underset{\Cap}{\varepsilon}}\ \approx\ |\mathcal{A}|^{\left(-\frac{h_+-k}{2}\right)}\tag{9}$$

with maximum extractable key length $\max(k)$:

$$\left\lfloor h_+-\frac{2}{\ln|\mathcal{A}|}\ln\Big(\frac{1}{|\mathcal{A}|}\lim_{n_t\uparrow\infty}\frac{1}{n_t}\big(\|\mathbf{T}_{KG}-\mathbf{T}_{KG}^*\|_{\mathcal{L}(L^p(\gamma))}\big)^{\otimes n_t}\Big)\right\rfloor.$$

Proof. Cauchy-Bunyakovsky-Schwarz inequality on Hilbert-Schmidt. [1]. ♡.

Theorem 1.2 (optimal). *For general* $\mathbb{E}^{gen}[\mathbf{\Gamma}]$, *resp. optimal* $\mathbb{E}^{opt}[\mathbf{\Gamma}]$,

$$\mathbb{E}^{gen}[\mathbf{\Gamma}]\ \geqslant\ \big(\mathbb{E}^{opt}[\mathbf{\Gamma}]\big)^2\ \textit{i.e.}\ \mathbb{E}^{opt}[\mathbf{\Gamma}]\leqslant\sqrt{\mathbb{E}^{gen}[\mathbf{\Gamma}]}\,.\tag{10}$$

Proof. See [1]. ♡.

Theorem 1.3 (quantum tripartite). *Finite* $\mathcal{X}$ *combinatorial formulation [4] subforest-tangle* ε*-rescaling* $T_1\otimes_\varepsilon T_2$, *Fig. 3,* $\forall\,h_{\min}(X|Q)\geqslant h_+$, *gives*

$$\frac{1}{|\mathcal{A}|}\big\|\mathbf{T}_{KGQ}-|\mathcal{A}|^{-k}\,\mathbf{I}_K\otimes\mathbf{T}_{GQ}\big\|_{\mathcal{L}(L^1(\gamma))}\ \leqslant\ \underset{(0,1]}{\underset{\Cap}{\varepsilon}}\ \approx\ |\mathcal{A}|^{\left(-\frac{h_+-k}{2}\right)}.\tag{11}$$

Proof. See [1]. ♡.

Parameter Estimation Conclusions

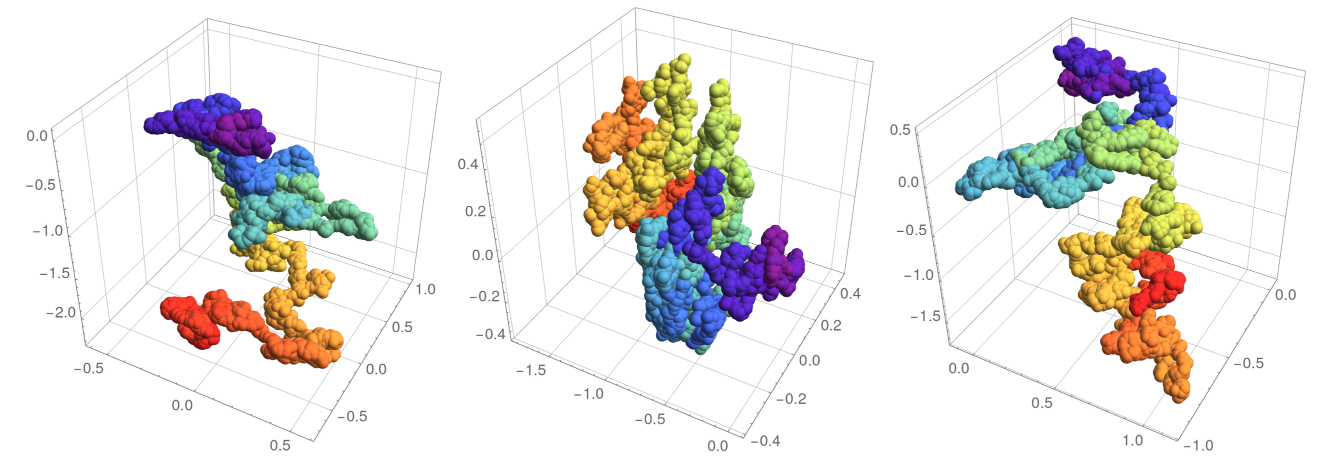

Fig. 4: $\mathbb{R}^3$ pairwise-key ε-subforest partition $\cong$ torus $\mathbb{S}^2$ Hopf fibration ring of blocks.

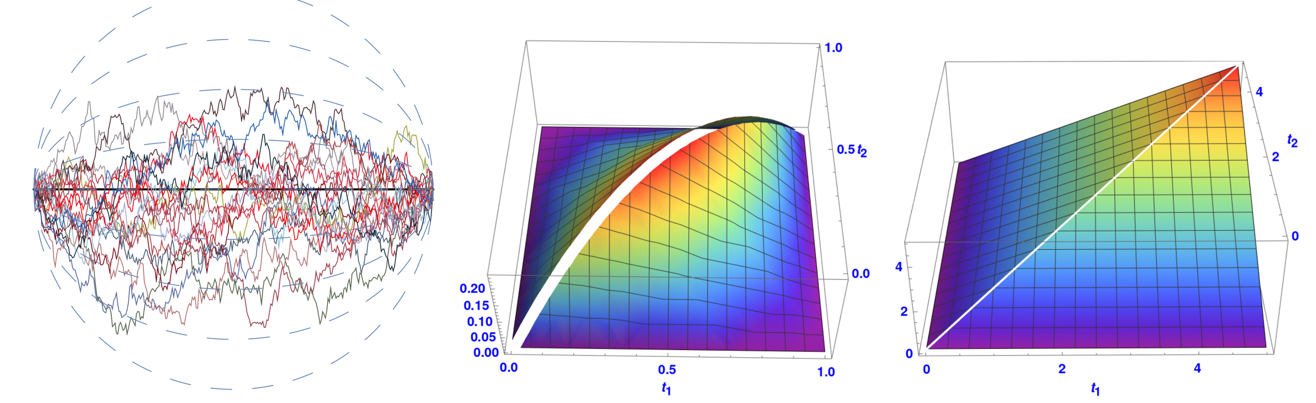

Fig. 5: Interpolated subforest-valued $(X_t)_{t\geqslant 0}$; weighted $\mathbb{E}[(x(t)-\mu(t))\otimes(x(s)-\mu(s))]$

For kronecker product $\otimes$, all tensor of column vector and row vector, resp. tensor of matrices, with volatility drift by uniform limit of Lemma 1.1; [1]; for $|\mathcal{A}|^m=10^k$, $|\mathcal{A}|^m\times|\mathcal{A}|^m\longrightarrow\exp(\alpha|\mathcal{A}|^{2m})$, on $|\mathcal{A}|^m$-block ($\mathbb{S}^{|\mathcal{A}|^m-1}$ ball) timeline intervals, $|\mathcal{A}|^m$-tuple random element interpolation, the Bernard et al., 2019, [2, 3], constant complexity $\mathcal{O}(1)$ convergent limit weight evolution follows a general pattern: increasing on interval $0\leqslant t\leqslant T/2$, decreasing on $T/2\leqslant t\leqslant T$; requiring smooth kernel distribution, variance relaxation over time, and addition of heavier tails for auxiliary (max or min non-ergodic part).

Future Research

Generalize simulations to higher genus $g\gg 0$ with optimization parameters.

References

[1] M. Belova and M. Bernard. *Asymptotic shape of quantum Markov semigroups for compact uniform trees.* arXiv:2012.00880, 2020.

[2] M. Bernard, V.A. Guskov, M.G. Ivanov, A.E. Kalugin, and S.L. Ogarkov. *Nonlocal scalar quantum field theory: functional integration, basis functions representation and strong coupling expansion.* Particles, 2(3):385–410, 2019.

[3] I.V. Chebotarev, V.A. Guskov, S.L. Ogarkov, and M. Bernard. *S-matrix of nonlocal scalar quantum field theory in basis functions representation.* Particles, 2(1):103–139, 2019.

[4] V. Jones. *A polynomial invariant for knots via von Neumann algebras.* Bull. Amer. Math. Soc. (N.S.), 12:103–111, 1985.

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